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of capricious adventures. We thus find the termination of the Romantic, in general, to consist in the accidentality both of the external and of the internal, and with this termination the two elements fall asunder. With this we emerge from the sphere of art altogether. It thus appears that the necessity which urges consciousness on to the attainment of a complete comprehension of the truth demands higher forms than Art is able in anywise to produce.

STATEMENT AND REDUCTION OF SYLLOGISM.

BY GEORGE BRUCE HALSTED.

At the basis of Logic stands the conception of class, the formation of general notions, the use of a word to denote the objects possessing a common attribute — *e. g.*, Geometer.

This assumes our ability to contemplate these objects apart from any or all others.

From this group we may select again those of them which belong to some other defined class, and so on.

We will represent classes by letters of the alphabet. Suppose x represents "men," and z , "geometers." Then xz would naturally be read "men geometers," and will mean such individuals of the class men as belong also to the class geometers. By an easy extension we take in all adjectives.

Suppose y means French, or the class French things; then xyz will mean all Frenchmen who are geometers.

We see that the order in which we select the classes is indifferent, in the sense that it gives the same result whatever order is taken.

In our example, if we first select geometers, then men, then French, we see that our final result, zxy , geometers who are Frenchmen, or yzx , French geometers who are men, should give the same final class of individuals. In ordinary language we use the position of words sometimes as a help toward expressing our meaning, but in this notation for Logic such help

is not needed. If we represent by x and z any two classes, known or unknown, we may generalize this law of the combination of classes, and express it by writing $xz=zx$.

We use the sign of equality to express, in the most general way, identity of individuals, coexistence of qualities, or equality of numbers.

The above equation, then, expresses the fact that logical multiplication is Commutative.

In using the word multiplication, and, further on, other terms of the common algebra of number, it is not even necessary to claim the slightest analogy between numerical multiplication and our process of logical combination. We are completely justified simply because their symbolic expressions are the same and subject to the same formal law.

But suppose we take as a class "all things," or the universe, or, much better, "the universe of discourse," and combine this with any other class, as x . We see that this does not change x in value; that $ux=xu=x$, whatever x may be. Have we in the common algebra of number a symbol possessing *formally* this property? This leads us to represent the universe of discourse by unity, by the simple arithmetical figure 1.

Again, it is indifferent whether from a group of objects considered as a whole we select the class x , or whether we divide the group into two parts, select the x 's from them separately, and then connect the results in one aggregate conception. That is, $x(y+z)=xy+xz=(y+z)x$, or logical multiplication is doubly distributive with reference to addition.

All substances outside of, or not belonging to or included in, a class may be considered together as forming another class, which is the negative of the first. In logic, by not- x or non- x we mean all the universe except x . The negative is a remainder, gained by the subtraction of the positive from the universe; x and non- x are the opposites under a given universe. So the term "not-gold" will apply to everything in the universe *except* what is truly gold. Representing "except" by the minus sign, we have, for example, non-gold=everything except gold= $1-g$, and so with any class.

Suppose, now, we combine any class with its negative, what result do we obtain? If b is taken to mean birds, what does $b(1-b)$ mean?

Every one immediately recognizes that it is impossible for any being at the same time to possess a quality and not to possess it. One or the other must be true; both cannot be true at the same time. So, whatever quality really defines the class "birds," there cannot be in the universe a single individual which at the same time really possesses it and does not possess it. There is nothing which is at the same time a bird and not a bird. To represent "nothing" we have the very convenient symbol O , nought. So we are driven to the conclusion that $b(1-b) = O$, and so of any class.

We proceed to the consideration of logical addition and subtraction. To express the aggregate conception of a group of objects consisting of partial groups, we use the conjunctions "and," "or," etc. — *e. g.*, "ladies and gentlemen," "men or women." In popular language these terms, "and," "or," etc., are often ambiguous. As ordinarily used, we cannot always tell whether they are meant to connect terms mutually exclusive or not. If we say a thing is either x or z , this mere statement does not explicitly inform any one whether we mean "if the one, then not the other," or only "if not the one, then the other;" whether we mean " x or z , but not both," or only " x or z , or, it may be, both." In our symbolic language we are able to avoid this vagueness perfectly. We supply the place of these words by the sign $+$, plus, which shall always mean that the classes which it connects are quite separate, entirely distinct, so that no member of one is found in another. Thus, "either a or b " in the first or exclusive sense is represented by $a(1-b) + b(1-a)$, and in the second or non-exclusive sense by $a + b(1-a)$. Here, again, as in the case of logical combination or multiplication, we see that we obtain the same aggregate group in whatever order the terms are taken — *e. g.*, if " a " represents chickens and " b " stands for ducks, then $a + b = b + a$, with the implication that no chickens are ducks.

The inverse operation to logical addition, or collecting parts

into a whole, is logical subtraction, or separating a part from a whole. This, as we have seen, is expressed in common language by the word "except," which we represent by the minus sign ($-$). Here, again, it is indifferent whether we express the excepted case first or last, or in what order we write any series of terms, some of which are affected by the minus sign — *e. g.*, "all plane figures, except triangles," means the same as, "excepting triangles, all plane figures." That is, $x-z = -z+x$. Moreover, if we make a selection by combining the adjective equilateral with the class "plane figures, excepting triangles," we reach the same result as if we combined it first with the class "plane figures," then with the class "triangles," and then took their difference. That is, $y(x-z) = yx-yz$; which shows that multiplication is distributive as well for a difference as for a sum. Applying the foregoing results to logical equations, we arrive immediately at the three general axioms (in which we use the word "equal" in its broadest sense, as signifying identity of individuals, coexistence of qualities, or equality of numbers):

1. If equal things are added to equal things, the wholes are equal.
2. If equal things are taken from equal things, the remainders are equal.
3. If equal things are multiplied by equivalents, the results are equal.

Hence we may add, subtract, or multiply logical equations, and transpose terms exactly as in the common algebra of number.

We are now ready to return to the equation given when we combine any class with its contrary or negative. In every such case we get $x(1-x) = 0$, because it is impossible for anything to possess a quality and not to possess it at the same time. But from $x(1-x) = 0$, since multiplication in logic is always distributive, we get, inevitably, $x-x^2 = 0$; and transposing, $x = x^2$. As x was entirely unrestricted, this law must hold for every logical term; and here, at last, we have something without a parallel in ordinary algebra. Instead of every number fulfilling this requirement, it is true for only two, namely, 1 and 0.

This peculiar law interferes essentially with *division*¹ in our notational algebra, which, as a logical operation, is identical with what is commonly called Abstraction.

But to return. If we always have $x=x^2$ xx , what is its interpretation in this form? Simply that if we combine a logical class with itself, or from a class select those members which it has in common with itself, the result is the class itself unchanged.

In passing from terms and their combination to the expression of propositions, we premise that if a proposition is negative we attach the negative particle to the predicate, and we denote "Some" by the indefinite symbol v . It will be convenient to apply the epithets of logical quantity, "universal" and "particular," and of logical quality, "affirmative" and "negative," to the *terms* of propositions, and not to the propositions themselves. There are, then, four classes of terms, namely: the universal-affirmative, "all x 's:" the particular-affirmative, "some x 's," or " x 's;" the universal-negative, "all non- x 's;" the particular-negative, "some non- x 's."

The expression "no x 's" is not properly a *term* of a proposition, for the meaning of the proposition "no x 's are y 's" is "all x 's are non- y 's." The subject of that proposition is, therefore, universal affirmative; the predicate, particular-negative.

By the various combinations of the four classes of terms, each with all, retaining the distinction always made in ordinary logics between subject and predicate, sixteen propositions will result. For it will be seen that we have four possible distinct subjects, in treating apart the term and its negative or complementary, x and $1-x$, which latter, for the sake of exhibiting symmetry, we will represent by $\bar{x}$.

Our four distinct subjects, then, are x , $\bar{x}$, vx , $v\bar{x}$; and we have as many distinct predicates, namely, y , $\bar{y}$, vy , $v\bar{y}$. Com-

¹ The extraordinary difficulties connected with a rigid and general exposition of algorithmic division, its limitations here, and the true reasons for them, can only be appreciated by one who has worked on that subject. It will require a separate paper, to which this may be taken as introductory.

binning these as above indicated, the sixteen propositions which result are as follows :

1. $x = vy$. All x 's are y 's.
2. $x = v\bar{y}$. No x 's are y 's.
3. $vx = vy$. Some x 's are y 's.
4. $vx = v\bar{y}$. Some x 's are non- y 's.
5. $\bar{x} = vy$. All non- x 's are y 's.
6. $\bar{x} = v\bar{y}$. No non- x 's are y 's.
7. $v\bar{x} = v\bar{y}$. Some non- x 's are non- y 's.
8. $v\bar{x} = vy$. Some non- x 's are y 's.
9. $vx = \bar{y}$. Some x 's are all the y 's.
10. $vx = y$. Some x 's are all the non- y 's.
11. $v\bar{x} = \bar{y}$. Some non- x 's are all the y 's.
12. $v\bar{x} = y$. Some non- x 's are all the non- y 's.
13. $\bar{x} = \bar{y}$. All non- x 's are all the y 's.
14. $x = \bar{y}$. All x 's are all the non- y 's.
15. $x = y$. All x 's are all the y 's.
16. $\bar{x} = y$. All non- x 's are all the non- y 's.

Ordinary Syllogism is inference from two propositions called the premises, having a common term called the middle term.

By the various combinations of the sixteen propositions, each with all, 256 pairs of premises will result. For brevity, these are not given, but, if needed, can be written out from the sixteen propositions already enumerated.¹

Now, as a first exercise for our Algorithmic Logic, as so far developed, let us apply it to the reduction or solution of these 256 Categorical Syllogisms.

In some sense, it was the perception of some parts of this problem of Syllogism, and of the need for solution, explanation, or reduction, which probably called logic into being. As Professor Bain says in *Mind*, January, 1878: "The meaning of Syllogism, then, is the *formal relation* between

¹ The 256 are given in tabular form, though in a somewhat different notation, on page 89 of Volume VIII of this journal, in a short article "On Logic," to which our attention was kindly called by the Editor. On page 90, also, are well stated, empirically, some conclusions which here we will apodeictically demonstrate.

the premises and the conclusion, whatever the matter be. If all syllogisms — all cases of argument or inference — were of the type Barbara, I doubt whether Syllogism would ever have been invented. Not that in Barbara there is not an element of form; but that being so easy, we need not even be conscious of it. But the inventor of the Syllogism was awakened to the fact that in many kinds of reasoning, not unfrequent in their occurrence, the formal relation of premises to conclusion was puzzling and uncertain, not to say misleading." Aristotle saw the need and value of a solution, and actually solved a considerable part accurately.

The general form in which we have stated the problem, in which every possible case is taken account of, gives us 256 pairs of premise-propositions; which would seem to make the complete discussion of Categorical Syllogism a matter of dreadful complication. In truth, without the application of mathematical ideas, it must have remained annoyingly intricate. But the result of an analytic solution, however tedious, may often be given synthetically in a very compendious form, and such is the nature of the Reduction of Syllogism which we now present.

Naturally we take it first in its elements — the propositions.

Now, by the simple consideration that as perfectly expressed in our notation every proposition is convertible — may be read backwards as truly as forwards — we see that six of the sixteen propositions (8–13) disappear, since in them no new relation between classes is given.

Against the last three propositions, against any universal substitutive judgment, it has been often and strongly urged that such are not logically simple propositions. The latest and best brief statement on this point is given in the January number of *Mind*, by the editor, in a short criticism of Professor Jevons. For those who still consider 14, 15, 16 as simple propositions, we can finish with them in a word.

In any pair of premises one of which is a universal substitutive, since this declares that one class or letter is exactly another, neither more nor less, read this other in place of the middle term and you have the conclusion.

We now have left only the seven simple logical propositions

or relations. Noting the symmetry, using the complementary classes, and observing that, whether we have x , or $\bar{x}$, the symmetry of relation is not altered, we say that all seven propositions can be brought under the two forms $XY=0$, $XY>0$; where the first expresses a relation of total exclusion, the second a relation of partial inclusion, between two classes. The relation $XY=0$, if both classes are taken positively, becomes $xy=0$, and may be read in any of the forms. No x 's are y 's; no y 's are x 's; all x 's are not- y 's; all y 's are not- x 's.

Similarly the relation $xy>0$ may be read in either of the forms: some x 's are y 's; some y 's are x 's. In each of the forms $XY=0$, $XY>0$, X may be taken to represent x or $\bar{x}$, and Y to represent y or $\bar{y}$. These two, then, are the only kinds of simple relations; it being understood that $\bar{x}$ may be substituted for X , or $\bar{y}$ for Y ; so that the example $\bar{x}y=0$ (all y 's are x 's) is the same kind of relation as $xy=0$; and $\bar{x}y>0$ (some y 's are not- x 's) is the same kind of relation as $xy>0$.

All propositions which have either the subject or predicate unaffected by the symbol v , "some," can be brought under the first form; the remaining propositions fall under the second form.

Now, the premises of every categorical syllogism are two propositions having a common term. Taking X and Z for the extremes, and Y for the middle term, the only combinations of premises are:

1. $XY=0$, $ZY=0$.
2. $XY=0$, $ZY>0 \therefore \bar{X}Z>0$.
3. $XY>0$, $ZY>0$.
4. $Xy=0$, $Z\bar{y}=0 \therefore XZ=0$.
5. $Xy=0$, $Z\bar{y}>0$.
6. $Xy>0$, $Z\bar{y}>0$.

And of these there are only two which give rise to a conclusion or relation between the extreme terms.

As regards the negative cases, this is at once seen to be so; thus, $xy=0$, $zy=0$ (no x 's are y 's, no z 's are y 's), leads to no conclusion in regard to the positive terms, x , z .

As regards the positive cases the conclusions may be easily proved to be valid by general symbolical reasoning. Thus, whatever Y may be, we know $Y = Yx + Y\bar{x}$; but in case 2, since $XY = 0 \therefore Y = Y\bar{x}$. But also we are given $ZY > 0$, therefore substituting, we have $ZY\bar{x} > 0 \therefore Z\bar{x} > 0$.

Again, $XZ = XZ\bar{y} + XZy$ always; but in 4 a factor of XZy , namely, Xy , is equal to nought, and a factor of $XZ\bar{y}$, namely, $Z\bar{y}$, is equal to nought; therefore, $XZ = 0 + 0 = 0$.

The logical signification of each step is obvious.

Still further, these two forms, the only forms which give a conclusion, differ only in the quantity of Z , which does not affect the reasoning, formally considered, in the slightest, since one of the original terms, say Z or vZ , may be allowed to enter unchanged into the conclusion. The seeming difference of form where in the second premise the y 's seem to be of different quality, as do the x 's in the conclusion, is produced by the fact that in translating $ZY > 0$ into an equation, the quality of Y is unchanged, for it gives "some Z 's are Y 's, $vZ = vY$; while in translating $Z\bar{y} = 0$ into like form, the quality of $\bar{y}$ is changed, since it gives "all Z 's are y 's," $Z = vy$. This is a particular case used as a diagram in the general demonstration, enabling us to see that if Δ is used to represent z and vZ , the general form to which we have apodeictically reduced all categorical Syllogism may be written,

$$X = v\bar{Y}; \Delta = vY; \therefore \Delta = v\bar{X}.$$

Thus we have demonstrated that a conclusion can be reached in every instance where two or more of the four terms contained in the two premises are universal, and that, too, whatever be the variation of the terms as to quality. When but one of the four terms is universal, a conclusion can be reached in all cases (*and in those only*) where the universal term is the middle term in one of the propositions, and the middle term in the other proposition is of the same quality — that is, positive when the universal term is positive, and negative when the universal term is negative, or where the propositions can be reduced to that form.